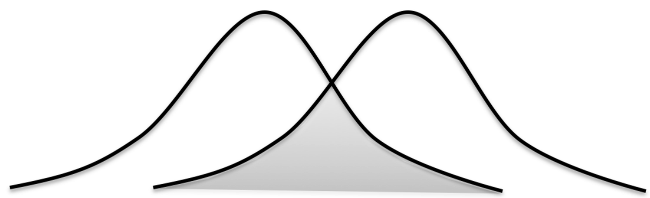
1 Appendix A

The following section derives the model used in the paper to define the rate of change of the bargaining space between two political parties as a function of their relative means.

As the two political parties become more ideologically separate and more homogenous, the bargaining space between the parties changes in predictable ways. This is defined as the area under the two curves at the point of their intersection.

Figure 1: Hypothetical Bargaining Space



Assuming that party ideology is distributed normally, we can find the area under the intersection of the two curves (the bargaining space, referred to here as B.S.) using the following formula:

$$B.S. = \int_{-\infty}^{int} \frac{1}{2\sqrt{\pi}\sigma_1} e^{-\frac{(x-\mu_1)^2}{2\sigma_1}} + \int_{int}^{\infty} \frac{1}{2\sqrt{\pi}\sigma_2} e^{-\frac{(x-\mu_2)^2}{2\sigma_2}} \tag{1}$$

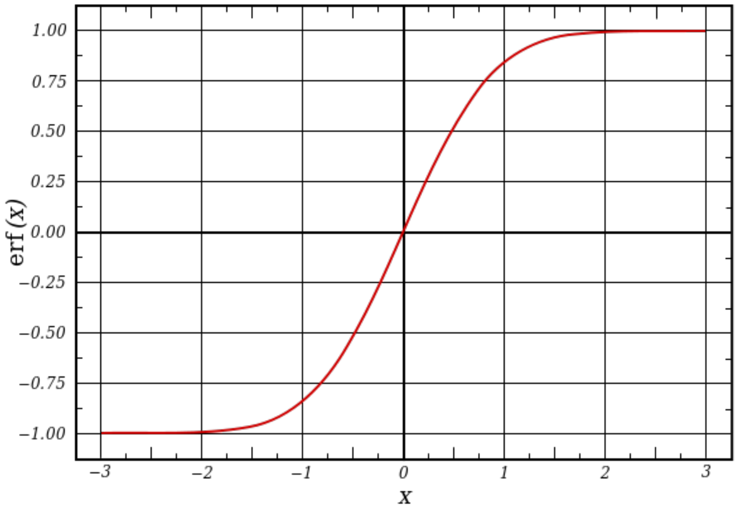
where *int* is the point at which the two curves intersect.

Because we are interested in the effect of changes in the mean, to simplify the math let us assume that the distribution is gaussian, and therefore its variance is 1. We can relax this assumption later to explore how changes in the variance (and, therefore, how changes in the ideological homogeneity of the parties) affect the area under the curve as well. This gives us a relatively simple formula:

$$B.S. = \int_{-\infty}^{int} \frac{1}{2\sqrt{\pi}} e^{-\frac{(x-\mu_1)^2}{2}} + \int_{int}^{\infty} \frac{1}{2\sqrt{\pi}} e^{-\frac{(x-\mu_2)^2}{2}} \tag{2}$$

The integral of an indefinite gaussian function does not have an indefinite solution— only a definite integral of a gaussian function can be evaluated. Integrating the *PDF* of a normal distribution results in the error function, $erf(x)$, which is graphically depicted in Figure 2:

Figure 2: Graph of Error Function



As we can observe, initial changes in x result in small differences in the area under the curve, but as x increases, the area increases dramatically, until we reach the other end of the bell curve and the rate declines again.

To find the rate of change of $erf(x)$ (the bargaining space), we take the derivative of the two integrals in Equation 2, which is simply:

$$\Delta_{B.S} = \frac{1}{2\sqrt{\pi}} e^{-\frac{(x-\mu_1)^2}{2}} + \frac{1}{2\sqrt{\pi}} e^{-\frac{(x-\mu_2)^2}{2}} \tag{3}$$

In this problem, x is best defined as the point of intersection *int*, which we can solve for by setting the two curves equal to each other and solving for x.

$$\frac{1}{2\sqrt{\pi}\sigma_1} e^{-\frac{(x-\mu_1)^2}{2\sigma_1}} = \frac{1}{2\sqrt{\pi}\sigma_2} e^{-\frac{(x-\mu_2)^2}{2\sigma_2}} \tag{4}$$

Again assuming that $\sigma = 1$:

$$\begin{aligned}
\frac{1}{2\sqrt{\pi}} e^{-\frac{(x-\mu_1)^2}{2}} &= \frac{1}{2\sqrt{\pi}} e^{-\frac{(x-\mu_2)^2}{2}} \\
e^{-\frac{(x-\mu_1)^2}{2}} &= e^{-\frac{(x-\mu_2)^2}{2}} \\
-\frac{(x-\mu_1)^2}{2} &= -\frac{(x-\mu_2)^2}{2} \\
(x-\mu_1)^2 &= (x-\mu_2)^2 \\
x^2 - 2\mu_1 x + \mu_1^2 &= x^2 - 2\mu_2 x + \mu_2^2 \\
2\mu_2 x - 2\mu_1 x &= \mu_2^2 - \mu_1^2 \\
x &= \frac{\mu_2^2 - \mu_1^2}{2\mu_2 - 2\mu_1}
\end{aligned}$$

Therefore, we arrive at the equation for the point of intersection:

$$int = \frac{\mu_2^2 - \mu_1^2}{2\mu_2 - 2\mu_1} \tag{5}$$

Substituting *int* for x in Equation 3, we develop the following equation for the rate of change of the bargaining space:

$$\begin{aligned}
\Delta_{B.S.} &= \frac{1}{2\sqrt{\pi}} e^{-\frac{1}{2}(\frac{\mu_2^2-\mu_1^2}{2\mu_2-2\mu_1}-\mu_1)^2} + \frac{1}{2\sqrt{\pi}} e^{-\frac{1}{2}(\frac{\mu_2^2-\mu_1^2}{2\mu_2-2\mu_1}-\mu_2)^2} \\
&= \frac{1}{2\sqrt{\pi}} e^{-\frac{1}{2}(\frac{\mu_2^2-\mu_1^2}{2\mu_2-2\mu_1}-\frac{\mu_1\mu_2-2\mu_1^2}{2\mu_2-2\mu_1})^2} + \frac{1}{2\sqrt{\pi}} e^{-\frac{1}{2}(\frac{\mu_2^2-\mu_1^2}{2\mu_2-2\mu_1}-\frac{2\mu_2^2-\mu_1\mu_2}{2\mu_2-2\mu_1})^2} \\
&= \frac{1}{2\sqrt{\pi}} e^{-\frac{1}{2}(\frac{\mu_2^2-\mu_1\mu_2+\mu_1^2}{2\mu_2-2\mu_1})^2} + \frac{1}{2\sqrt{\pi}} e^{-\frac{1}{2}(\frac{\mu_2^2-\mu_1\mu_2+\mu_1^2}{2\mu_2-2\mu_1})^2} \\
&= \frac{1}{\sqrt{\pi}} e^{-\frac{1}{8}(\frac{(\mu_2-\mu_1)^2}{\mu_2-\mu_1})^2} \\
&= \frac{1}{\sqrt{\pi}} e^{-\frac{1}{8}(\mu_2-\mu_1)^2}
\end{aligned}$$

Therefore, the equation for the rate at which the bargaining space should change is an exponential function. Because the coefficient of the function will be determined by the poisson model used in Section 6 of the paper, we simplify the equation to the following model:

$$\Delta_{B.S.} = e^{-\frac{1}{8}(\mu_2-\mu_1)^2} \tag{6}$$